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L-moments: Analysis and Estimation of Distributions using Linear Combinations of Order Statistics

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SUMMARY

L-moments are expectations of certain linear combinations of order statistics. They can be defined for any random variable whose mean exists and form the basis of a general theory which covers the summarization and description of theoretical probability distributions, the summarization and description of observed data samples, estimation of parameters and quantiles of probability distributions, and hypothesis tests for probability distributions. The theory involves such established procedures as the use of order statistics and Gini's mean difference statistic, and gives rise to some promising innovations such as the measures of skewness and kurtosis described in Section 2, and new methods of parameter estimation for several distributions. The theory of L-moments parallels the theory of (conventional) moments, as this list of applications might suggest. The main advantage of L-moments over conventional moments is that L-moments, being linear functions of the data, suffer less from the effects of sampling variability: L-moments are more robust than conventional moments to outliers in the data and enable more secure inferences to be made from small samples about an underlying probability distribution. L-moments sometimes yield more efficient parameter estimates than the maximum likelihood estimates.

Keywords: ESTIMATION; HYPOTHESIS TESTING; KURTOSIS; L-STATISTICS; MOMENTS; ORDER STATISTICS; SKEWNESS

1. INTRODUCTION

It is standard statistical practice to summarize a probability distribution or an observed data set by its moments or cumulants. It is also common, when fitting a parametric distribution to a data set, to estimate the parameters by equating the sample moments to those of the fitted distribution. Yet moment-based methods, although long established in statistics, are not always satisfactory. It is sometimes difficult to assess exactly what information about the shape of a distribution is conveyed by its moments of third and higher order; the numerical values of sample moments, particularly when the sample is small, can be very different from those of the probability distribution from which the sample was drawn; and the estimated parameters of distributions fitted by the method of moments are often markedly less accurate than those obtainable by other estimation procedures such as the method of maximum likelihood.

The alternative approach described here is based on quantities which we call L-moments. These are analogous to the conventional moments but can be estimated by linear combinations of order statistics, i.e. by L-statistics. L-moments have the

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