A Universal Performance Measure

We present a new approach to analysing returns distributions, the Omega function, which may be used as a natural performance measure. Analysis based on Omega is in the spirit of the downside, lower partial moment and gain-loss literatures. The Omega function captures all of the higher moment information in the returns distribution and also incorporates sensitivity to return levels. We indicate how this may be applied across a broad range of problems in financial analysis and apply it to a range of hedge fund style or strategy indices.

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INTRODUCTION

Many of the difficulties we encounter in performance measurement and attribution are rooted in two over-simplifications. The first is that mean and variance fully describe the distribution of returns. The second is that the risk-reward characteristics of a portfolio may be described without reference to any return level aside from the mean return. It is a generally accepted fact of empirical finance that returns from investments are not distributed normally. Thus in addition to mean and variance, higher moments are required for a complete description. It is likewise clear that a return at the level of the mean may be regarded as a gain by one investor and as a loss by another and that the “risk” of a return far above the mean has a different impact than that of one far below the mean.

In this paper we introduce a performance evaluation measure, $\Omega$, which accomplishes the task of incorporating all of the higher moments of a returns distribution. It provides a full characterisation of the risk reward characteristics of the distribution in a way which is intuitively appealing and easily calculated. Instead of estimating any individual moments it measures their total impact, which is of course precisely what is of interest to practitioners. It also provides a risk-reward evaluation of a returns distribution which incorporates the beneficial impact of gains as well as the detrimental effect of losses, relative to any individual’s loss threshold.

Omega is a natural feature of the returns distribution. In fact it is, in a mathematically precise sense, equivalent to the returns distribution. Thus its construction from a returns distribution is entirely canonical, requiring no choices and introducing no ambiguity not already present in the returns data. Omega is a function that may be evaluated at any value in the range of possible returns, so that it allows performance comparisons with respect to any ‘risk’ threshold in this range.