Beyond Mean–Variance: Performance Measurement in a Nonsymmetrical World

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Most practitioners use the capital asset pricing model to measure investment performance. The CAPM, however, assumes either that all asset returns are normally distributed (and thus symmetrical) or that investors have mean–variance preferences (and thus ignore skewness). Both assumptions are suspect. Assuming only that the rate of return on the market portfolio is independently and identically distributed and that markets are "perfect," this article shows that the CAPM and its risk measures are invalid: The market portfolio is mean–variance inefficient, and the CAPM alpha mismeasures the value added by investment managers. Strategies with positively skewed returns, such as strategies limiting downside risk, will be incorrectly underrated. A simple modification of the CAPM beta, however, will produce correct risk measurement for portfolios with arbitrary return distributions, and the resulting alphas of all fairly priced options and/or dynamic strategies will be zero. The risk measure requires no more information to implement than the CAPM.

How can one determine whether an investment manager has added value to the funds the manager is handling relative to the risk the manager is taking? Correct performance assessment requires both good theory, to determine the proper measure of risk, and appropriate statistical techniques, to quantify risk magnitudes. This article focuses on measures of risk and the implications of those measures for investment performance evaluation.

Although some notable advances have recently been made in the theory of performance measurement, most practice is firmly rooted in the approach of the capital asset pricing model.[1] In the CAPM world, the appropriate measure of the risk of any asset or portfolio p is given by its beta:

$$\beta_p = \frac{\text{cov}(r_p, r_{mkt})}{\text{cov}(r_{mkt}, r_{mkt})} = \frac{\text{cov}(r_p, r_{mkt})}{\text{var}(r_{mkt})}, \quad (1)$$

where r_p and r_{mkt} are the random returns on, respectively, the portfolio and the market.

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In equilibrium, all assets and portfolios will have the same return after adjustment for risk, which implies the following formula for the expected return on the portfolio:

$$E(r_p) = r_f + \beta_p[E(r_{mkt}) - r_f], \quad (2)$$

where r_f is the risk-free interest rate.

Superior performance in the CAPM world is measured by alpha, which is the incremental expected return resulting from applying managerial information (e.g., stock selection or market timing). The portfolio alpha can be represented formally as

$$\alpha_p = E(r_p \mid M) - E(r_p) = E(r_p \mid M) - \beta_p[E(r_{mkt}) - r_f] - r_f, \quad (3)$$

where $E(r_p \mid M)$ is the expected return to the portfolio conditioned by the information used by the manager, M.[2] In the CAPM equilibrium, alphas will be zero unless a manager has superior information. A portfolio with a positive alpha offers an expected return in excess of its equilibrium risk-adjusted level and, in this sense, has superior performance. A related, but not identical, performance measure is the Sharpe ratio (SR) of a portfolio. In it, $SR_p = [E(r_p \mid M) - r_f]/\sigma_p$. The Sharpe ratio provides an appropriate measure of investor welfare when the investor has mean–variance preferences and invests exclusively in the portfolio (and perhaps a