# Stochastic Dominance, Mean Variance, and Gini's Mean Difference

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The two methods frequently used for comparing uncertain prospects are the mean variance ($MV$) and the stochastic dominance ($SD$) approaches.[1] The main idea of $MV$ is to represent the distribution of the uncertain prospects by two summary statistics: the mean, representing the reward, and the variance, representing the variability. The use of the summary statistics simplifies the analysis, and is amenable to geometric treatment, but as has been pointed out (see Giora Hanoch and Haim Levy; Michael Rothschild and Joseph Stiglitz), may lead to unwarranted conclusions. The $SD$ approach does not lead to unwarranted conclusions, but it is much more complicated, usually leads to large efficient sets, and is difficult to treat geometrically. Another disadvantage of the $SD$ approach is that, as far as I know, there is no algorithm for the construction of an efficient portfolio.

At first glance, it seems that the gap between the two different approaches cannot be bridged. Tobin argues that critics of the $MV$ approach:

> ...owe us more than demonstrations that it rests on restrictive assumptions. They need to show us how a more general and less vulnerable approach will yield the kind of comparative-static results that economists are interested in. This need is satisfied neither by the elegant but nearly empty existence theorems of state preference theory nor by normative prescriptions to the individual that he should consult his utility and his subjective probabilities and then maximize. [1969, p. 14]

On the other hand, Hanoch and Levy reach the conclusion that:

> The identification of riskiness with variance, or with any other single measure of dispersion, is clearly unsound. There are many obvious cases, where more dispersion is desirable, if it is accompanied by an upward shift in the locations of the distribution, or by an increasing positive asymmetry. [p. 344]

The aim of this paper is to present a new method for comparing uncertain prospects. It is based on using the mean and Gini's mean difference (henceforth, $MG$ approach) as the summary statistics to describe the distribution. It is shown for any two distributions $F$, $G$ that $\mu_F \geqslant \mu_G$ and $\mu_F - \Gamma_F \geqslant \mu_G - \Gamma_G$ are necessary conditions for first- and second-degree stochastic dominance (where $\mu$ is the mean and $\Gamma$ is half Gini's mean difference). These two necessary conditions yield only prospects that would be included in an $SD$ efficient set. The $MG$ approach enables us to construct portfolios which are $SD$ efficient. Since it uses two summary statistics, it is almost as simple as the $MV$ approach and like it has a simple geometrical presentation. In certain cases the $MV$ efficient set can be derived by the $MG$ approach. The $MG$ approach may thus be viewed as a compromise with some of the merits of the other two.

I begin by presenting the relationship between $SD$ and Gini's mean difference and go on to analyze the relationship between the efficient sets resulting from the $SD$, $MV$, and $MG$ approaches. I then use the last to construct efficient $SD$ portfolios.

## I. Gini's Mean Difference and Stochastic Dominance

Following Hanoch and Levy, I define two classes of utility functions $U_i$ ($i = 1, 2$) where

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[1] For a review of the $SD$ approach see Yoram Kroll and Haim Levy. The $MV$ approach is discussed by H. M. Markowitz and James Tobin (1965).